# ME 321: FLUID MECHANICS-I


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**Lecture-09**

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**Fluid dynamics**
- **Bernoulli Equation**


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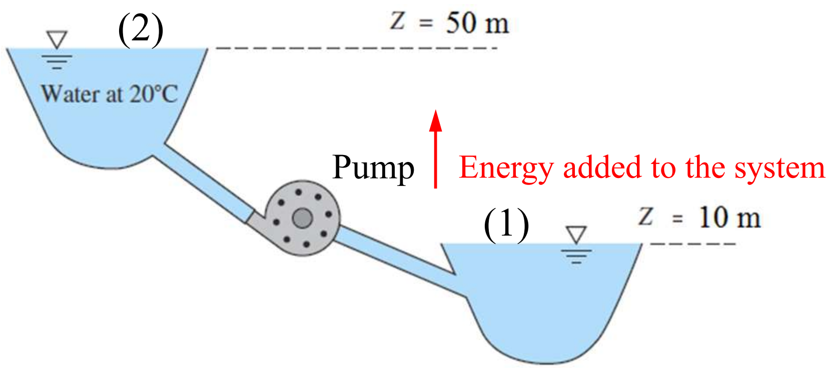
# Flow system with turbomachinery

Modified Bernoulli equation i.e. the **energy equation** in a **flow system with pump**:

**No head loss:** $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_P=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$




$h_P$ = head (energy) added to the system

$P_{pump}=\gamma Q h_P$ **(pump hydraulic power)**

Pump:
Input: Electrical power
Output: hydraulic/mechanical power

**With head loss:** $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_P=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

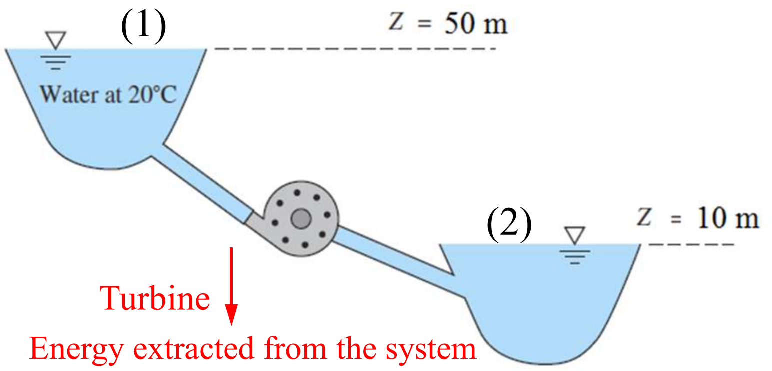
$$\eta=\frac{P_{out}}{P_{in}}=\frac{\gamma Q h_P}{P_{in}\,(\text{elect. power})}$$

$h_L$ = head loss (major/minor) to be added at downstream

$$h_L=K\frac{V^2}{2g}$$

# Flow system with turbomachinery

Modified Bernoulli equation i.e. the **energy equation** in a **flow system with turbine**:




**No head loss:**

$$\frac{p_1}{\gamma}+\frac{V_1^{\,2}}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^{\,2}}{2g}+z_2+h_T$$

$h_T$ = head (energy) extracted from the system

$$P_{turbine}=\gamma Q h_T$$ **(turbine hydraulic power)**

Turbine:

Input: hydraulic/mechanical power

Output: Electrical power

**With head loss:**

$$\frac{p_1}{\gamma}+\frac{V_1^{\,2}}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^{\,2}}{2g}+z_2+h_T+h_L$$

$$\eta=\frac{P_{out}}{P_{in}}=\frac{P_{in}\text{ (elect. power)}}{\gamma Q h_T}$$

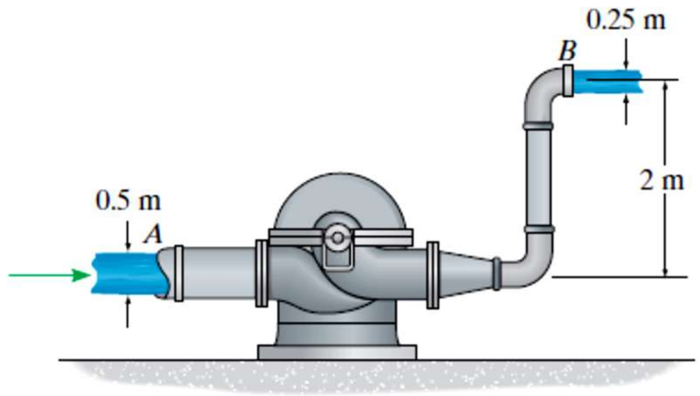
$h_L$ = head loss (major/minor) to be added at downstream

## Problem 8

The electrical power input to the pump is 10 kW. If the pump has an efficiency of 80%, and the increase in pressure from ***A*** to ***B*** is 100 kPa, determine the volumetric flow rate of water through the pump in cases of

(i) No head loss between A to B

(ii) Head loss between A to B is 1.25 m.




**Solution:**

(i) <u>No head loss between A to B:</u>

Bernoulli equation between points A and B for this case is-

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_p = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \qquad (i)$$

**; $h_p$ is the head developed by the pump**

From continuity equation:

$$Q = v_A A_A = v_B A_B \qquad \text{(unknown)}$$

$$\Rightarrow Q = v_A\left(\frac{\pi}{4}d_A^2\right) = v_B\left(\frac{\pi}{4}d_B^2\right)$$

$$\Rightarrow Q = v_A\left(\frac{\pi}{4}0.5^2\right) = v_B\left(\frac{\pi}{4}0.25^2\right)$$

$$\Rightarrow v_A = 5.09Q \qquad \& \qquad v_B = 20.37Q$$

# Problem 8

For the pump

$$\eta = \frac{P_{out}}{P_{in}} \left( \frac{\text{Hydraulic power output}}{\text{Electrical power input}} \right)$$

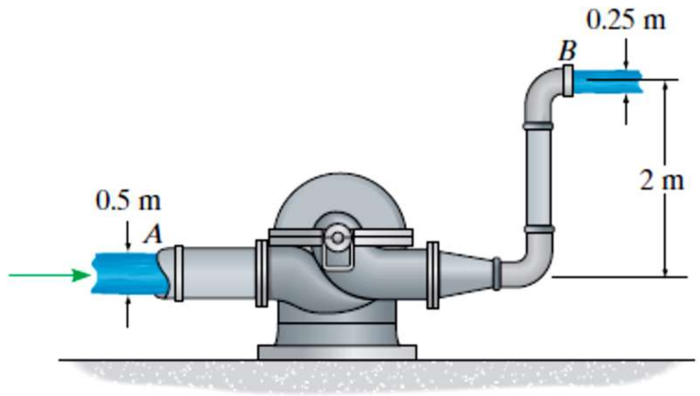
$$\Rightarrow 0.8 = \frac{P_{out}}{10 \times 10^3}$$

$$\Rightarrow P_{out} = 0.8 \times 10 \times 10^3$$

$$\Rightarrow \gamma Q h_P = 0.8 \times 10 \times 10^3$$

$$\Rightarrow h_P = \frac{0.8 \times 10 \times 10^3}{\gamma Q}$$

$$\boxed{\Rightarrow h_p = \frac{0.8155}{Q}}$$




Now consider equation (i)

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_P = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \qquad (i)$$

$$\Rightarrow \frac{V_A^2}{2g} + 0 + h_P = \frac{p_B - p_A}{\gamma} + \frac{V_B^2}{2g} + 2$$

$$\Rightarrow \frac{(5.09Q)^2}{2g} + 0 + \frac{0.8155}{Q} = \frac{100 \times 10^3}{(1000 \times 9.81)} + \frac{(20.37Q)^2}{2g} + 2$$

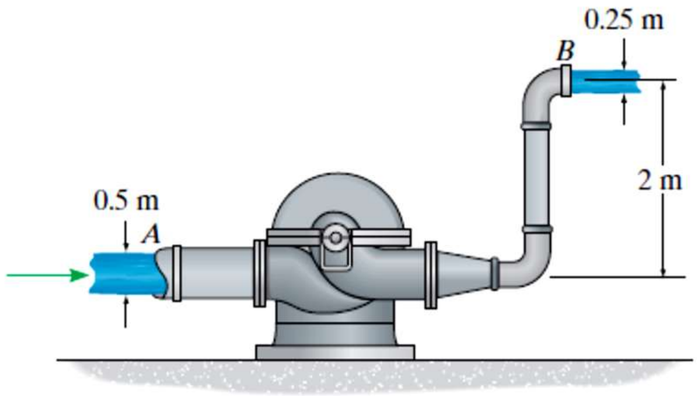
$$\boxed{\Rightarrow \frac{0.8155}{Q} = 12.19 + 19.83Q^2}$$

# Problem 8

On solving the last equation to get the flow rate, Q: (***through numerical solution***)

$$\Rightarrow \frac{0.8155}{Q} = 12.19 + 19.83Q^2$$

$$\boxed{\Rightarrow Q \approx 0.0664 \ \text{m}^3/\text{s} \quad (\equiv 239\,\text{m}^3/\text{hr},\ 66.4\ \text{l/s})}$$ **Ans. (i)**




(ii) <u>Head loss between A to B is 1.25 m:</u>

Bernoulli equation between points A and B for this case is-

$$\boxed{\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_P = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_L \qquad (ii)}$$

; $h_p$ **is the head developed by the pump**
$h_L$ **is the head loss from points A to B**

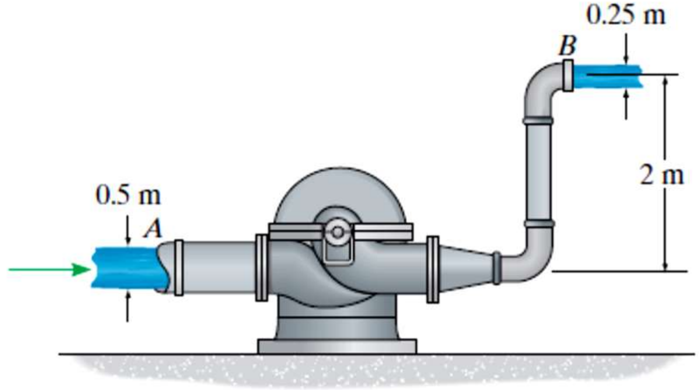
$$\Rightarrow \frac{V_A^2}{2g} + 0 + h_P = \frac{p_B - p_A}{\gamma} + \frac{V_B^2}{2g} + 2 + 1.25$$

$$\Rightarrow \frac{(5.09Q)^2}{2g} + 0 + \frac{0.8155}{Q} = \frac{100\times10^3}{(1000\times9.81)} + \frac{(20.37Q)^2}{2g} + 3.25$$

$$\boxed{\Rightarrow \frac{0.8155}{Q} = 13.44 + 19.83Q^2}$$

# Problem 8

On solving the last equation to get the flow rate, Q: (***through numerical solution***)




$$\Rightarrow \frac{0.8155}{Q} = 13.44 + 19.83Q^2$$

$$\boxed{\Rightarrow Q \approx 0.0604 \ \text{m}^3/\text{s} \quad (\equiv 218\,\text{m}^3/\text{hr},\ 60.4\ \text{l/s})}$$

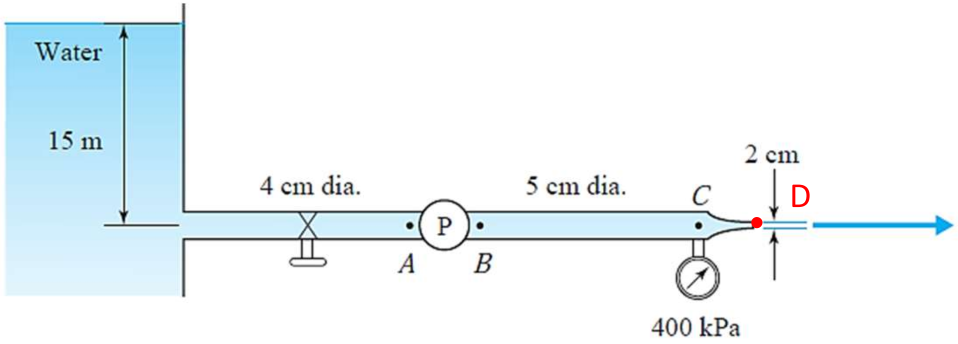
**Ans. (ii)**

**Volumetric flow rate will be reduced in case of head loss due to fluid friction (major loss) and pipe fittings (minor loss).**

**Head losses will be covered in detail in ME 323 (L3 T2)**

# Problem 9

Find the power requirement of the 85%-efficient pump shown in Fig. if the loss coefficient up to *A* is 3.2, and from *B* to *C*, *K*=1.5. Neglect the losses through the exit nozzle.
Also, calculate $p_A$ and $p_B$.




$$h_L = K\frac{V^2}{2g}$$

**Solution:**

$$Q_C = Q_D$$

$$\frac{\pi}{4}0.05^2\, V_C = \frac{\pi}{4}0.02^2\, V_D$$

$$\therefore V_c = 0.16\, V_D$$

Bernoulli equation between points C and D (across the nozzle) -

$$\frac{p_C}{\gamma} + \frac{{V_C}^2}{2g} + z_C = \frac{p_D}{\gamma} + \frac{{V_D}^2}{2g} + z_D + h_{L\,C-D}$$

$$\frac{400 \times 10^3}{9810} + \frac{(0.16V_D)^2}{2g} + 0 = 0 + \frac{{V_D}^2}{2g} + 0 + 0$$

$; h_{L\,C-D} = 0\ (no\ loss\ through\ the\ nozzle)$

$$\therefore V_D = 28.6\, m/s \qquad \therefore V_C = 4.6\, m/s$$

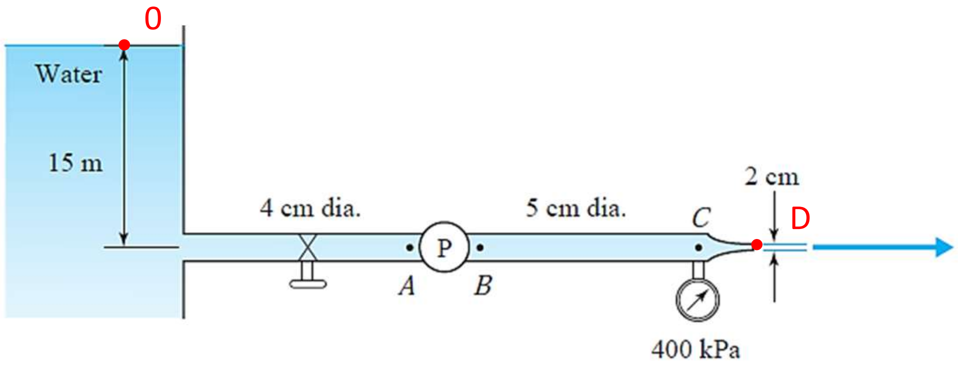
# Problem 9

Considering points B and C -

$$Q_B = Q_C$$
$$\therefore V_B = V_C = 4.6\ m/s$$

Considering points A and C -

$$Q_A = Q_C$$
$$\frac{\pi}{4} 0.04^2\ V_A = \frac{\pi}{4} 0.05^2 (4.6)$$
$$\therefore V_A = 7.2\ m/s$$




Bernoulli equation between points 0 and D (surface to exit) -

$$\frac{p_0}{\gamma} + \frac{V_0{}^2}{2g} + z_0 + h_p = \frac{p_D}{\gamma} + \frac{V_D{}^2}{2g} + z_D + h_{L\ 0-D}$$

$$0 + 0 + 15 + h_p = 0 + \frac{28.6^2}{2g} + 0 + 3.2\frac{7.2^2}{2g} + 1.5\frac{4.6^2}{2g}$$

$$; h_{L\ 0-D} = h_{L\ 0-A} + h_{L\ B-D} = K_A\frac{V_A^2}{2g} + K_B\frac{V_B^2}{2g}$$

$$\therefore h_p = 36.8\ m$$

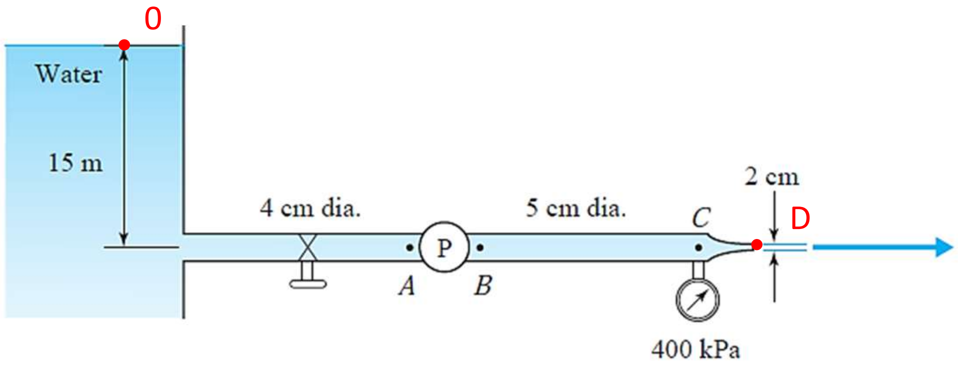
# Problem 9

Pump requirement to run the pump

$$P_{elect.} = \frac{\gamma Q h P}{\eta}$$

$$= \frac{(9810)(\frac{\pi}{4}\times 0.02^2 \times 28.6)(36.8)}{0.85}$$

= 3.82 kW (Ans.)



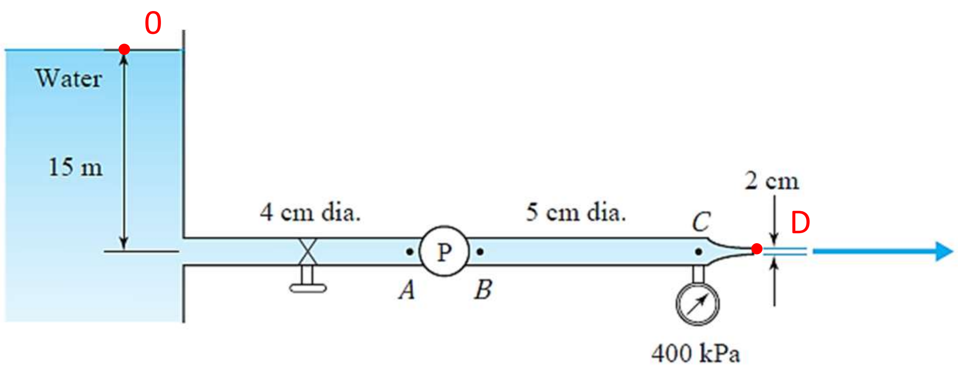

Bernoulli equation between points 0 and A (surface to A) -

$$\frac{p_0}{\gamma} + \frac{{V_0}^2}{2g} + z_0 = \frac{p_A}{\gamma} + \frac{{V_A}^2}{2g} + z_A + h_{L\,0-A}$$

$$0 + 0 + 15 = \frac{p_A}{\gamma} + \frac{7.2^2}{2g} + 0 + 3.2\frac{7.2^2}{2g}$$

$$\therefore p_A = 38.3\ kPa\ (Ans.)$$

# Problem 9




Bernoulli equation between points 0 and B (surface to B) -

$$\frac{p_0}{\gamma} + \frac{{V_0}^2}{2g} + z_0 + h_p = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + z_B + h_{L\,0-B}$$

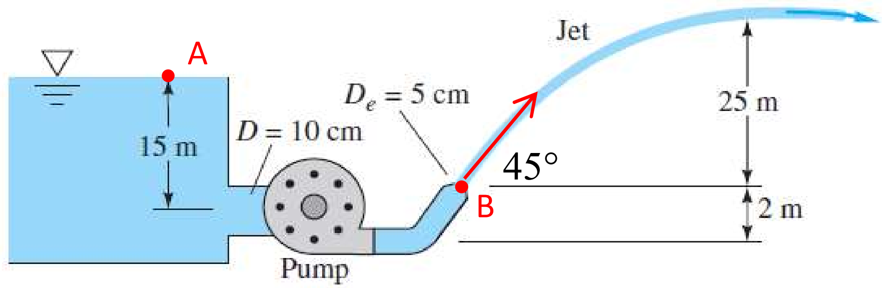
$$0 + 0 + 15 + 36.8 = \frac{p_B}{\gamma} + \frac{4.6^2}{2g} + 0 + 3.2\frac{7.2^2}{2g}$$

$$; h_{L\,0-B} = h_{L\,0-A} + h_{L\,A-B} = K_A\frac{V_A^2}{2g} + 0$$

$$\therefore p_B = 414.6\,kPa\,(Ans.)$$

# Problem 10

The pump in the figure creates a water jet oriented at 45° (to travel a maximum horizontal distance). System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?
How much electrical power is required to run this operation if pump efficiency is 65%?




**Solution:**

For vertical distance traveled by the jet is

$$V_f^2 = V_i^2 - 2gz$$

For **maximum vertical distance** traveled by the jet is

$$V_f = 0$$
$$V_i = V_B \sin 45°$$

$$\therefore V_B \sin 45° = \sqrt{2gZ_{\max}}$$

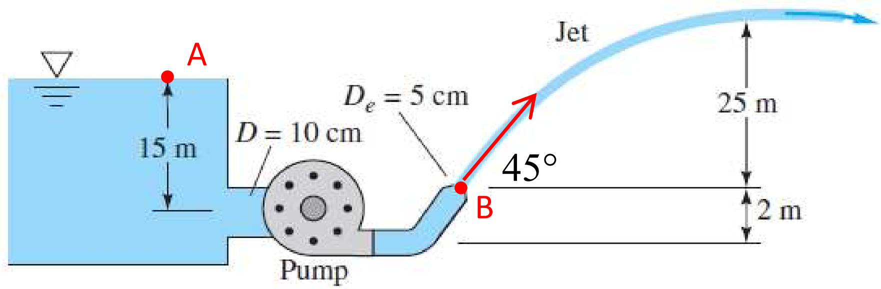
$$\therefore V_B = \frac{\sqrt{2gZ_{\max}}}{\sin 45°} = \frac{\sqrt{2(9.81)(25)}}{\sin \quad °} \approx 31.3 \text{ m/s}$$

# Problem 10

$$\frac{p_A}{\gamma}+\frac{V_A^{\,2}}{2g}+z_A+h_P=\frac{p_B}{\gamma}+\frac{V_B^{\,2}}{2g}+z_B+h_L$$

$$\Rightarrow 0+0+15+h_P=0+\frac{31.3^2}{2g}+2+6.5$$

$$\Rightarrow h_P=43.4\ \text{m}$$




Power must be delivered by the pump

$$P_{Pump}=\gamma Q h_P=(1000)(9.81)\left(\frac{\pi}{4}(5\times10^{-2})^2(31.3)\right)(43.4)$$

$$\Rightarrow P_{Pump}=26.2\ \text{kW}$$

Electrical power required for this operation

$$\eta=\frac{P_{out}}{P_{in}}=\frac{\gamma Q h_P}{P_{in}\ (\text{elect. power})}\qquad\Rightarrow 0.65=\frac{26.2}{P_{in}}$$

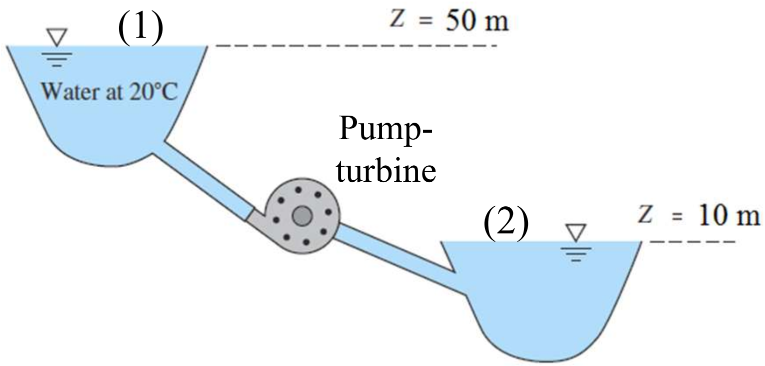
$$\Rightarrow P_{in}=40.3\ \text{kW}\qquad(\text{Ans.})$$

# Problem 11

The *pump-turbine* system in the figure draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoir to restore the situation. For a design flow rate of 50,000 lit/min in either direction, the friction head loss is 5 m.

Estimate the power in kW

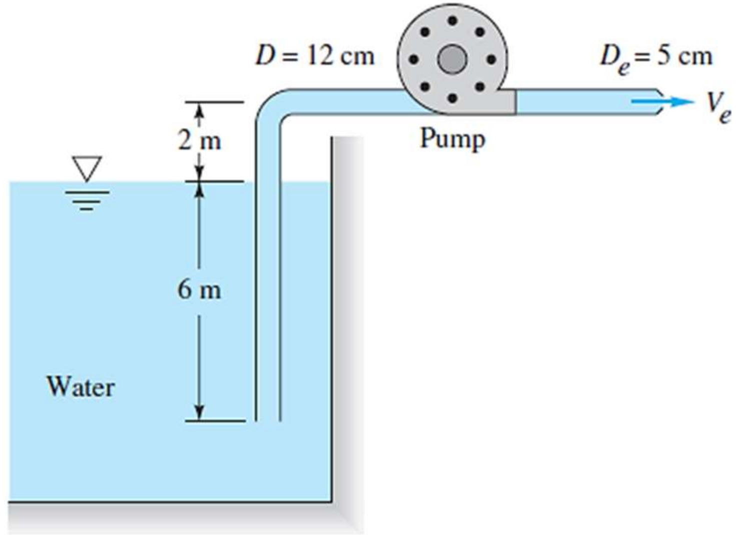
(a) extracted by the turbine and
(b) delivered by the pump.




**Solution:**

## Problem 12

When the pump in Fig. draws 220 $m^3/h$ of water at 20°C from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.



# Problem 13

Water flows from a reservoir through a 0.76 m-diameter pipeline to a turbine-generator unit and exits to a river that is 35 m below the reservoir surface. If the flow rate is 9200 $m^3$/hr, and the turbine-generator efficiency is 88%, calculate the power output.

Assume the loss coefficient in the pipeline (including the exit) to be $K$ = 2.